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# RESEARCH MEMORANDUM

APPLICATION OF NONLINEAR AERODYNAMIC  
STABILITY CHARACTERISTICS TO SIMPLIFY MISSILE  
CONTROL AND GUIDANCE SYSTEMS

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SUMMARY

A missile configuration having three stable trim positions which produce positive, negative, and zero lift has been discussed, and the use of this nonlinear stability characteristic as a possible means of simplifying the required control system has been considered. This missile is considered to operate at one of the three trim positions at all times, and the purpose of the control is to change to the desired trim position and, hence, the desired maneuvering. Two methods of controlling this on-off or flicker operation of the aerodynamics have been presented; namely, the one-pulse method, where a constant control-actuation time is used when a plus or minus control moment is applied to change trim positions, and the two-pulse method, where a control sequence of two pulses of opposite signs are employed. The simplification of the control system results because the servo operation or sequence can be preset and identical except for sense for all input commands. Analog results demonstrate the system operation in changing from one trim position to another. The use of such a system in a guidance situation has been discussed to illustrate that simplified guidance equipment can also be used.

INTRODUCTION

The simplification of missile control systems has been recognized as a prime requisite in increasing the operational reliability of guided missiles. However, it must also be realized that such simplification should not unduly impair system performance in the ultimate accomplishment of the desired task. Several investigations (refs. 1 and 2) have been directed toward particular features or ideas which should lead to less complex internal requirements for the control system.

The simplification of the control system might come in any number of

ways. In this presentation a simplification of the control system is accomplished through a novel use of nonlinear aerodynamic stability characteristics of a cruciform configuration. An aircraft capable of a positive, negative, and a zero lift characteristics is used, and a simple control impulse required to change trim positions is considered. The use of such an arrangement is also considered as a possible means of simplifying the required guidance equipment. The purpose of this investigation is to show how such a nonlinear system can be used so that the internal control system will be more rugged and more reliable and the system operation will be simplified. It is not intended that the missile configuration considered in this paper is an optimum one for such an application; it is only used to illustrate the idea. Additional aerodynamic development would be required before a completely acceptable configuration could be evolved for tactical consideration. This investigation was conducted primarily by using the Reeves Electronic Analog Computer (REAC) and the Electronic Analog Simulator Equipment (EASE). The phase-plane method of nonlinear mechanics (e.g., refs. 3 and 4) was also used to illustrate the conditions which lead to changes in trim positions.

## SYMBOLS

|                        |                                                                      |
|------------------------|----------------------------------------------------------------------|
| $\alpha$               | angle of attack, radians unless otherwise noted                      |
| $\alpha_{\text{trim}}$ | trim or steady-state angle of attack, radians unless otherwise noted |
| $\gamma$               | flight-path angle, deg                                               |
| $\sigma$               | angle between missile-target line of sight and reference, deg        |
| $\epsilon$             | error angle, $\sigma - \gamma$ , deg                                 |
| $\delta$               | control deflection angle, radians unless otherwise noted             |
| $\dot{\theta}$         | angular pitching velocity, radians/sec                               |
| $n_1$                  | command acceleration, g units                                        |
| $n_0$                  | missile acceleration, g units                                        |
| $\Delta t$             | control-actuation or dwell time, sec                                 |
| $M$                    | Mach number                                                          |

|               |                                                        |
|---------------|--------------------------------------------------------|
| $C_m$         | pitching-moment coefficient, $\text{Moment}/qS\bar{c}$ |
| $C_L$         | lift coefficient, $\text{Lift}/qS$                     |
| $t/c$         | ratio of maximum wing thickness to wing chord          |
| $q$           | dynamic pressure, $\text{lb}/\text{sq ft}$             |
| $S$           | total wing area, $\text{sq ft}$                        |
| $\bar{c}$     | wing mean aerodynamic chord, $\text{ft}$               |
| $C_m(\delta)$ | control moment coefficient                             |

### AERODYNAMIC CONTROL AND GUIDANCE METHOD

The operation of the systems of control and guidance can be briefly summarized as follows: A missile having three trim positions of angle of attack producing positive, negative, and zero lift is considered to operate at one of these positions at all times. The purpose of the control is to change from one trim position to another as desired. The operation of the missile in a positive-, negative-, or zero-maneuvering state is used with a guidance system which calls for one of these states depending on whether a positive, a negative, or a zero maneuver is required to bring the missile velocity vector in line with the target. Hence, the seeker operation is nonlinear and has a central dead zone where the missile would be approximately directed toward the target and two sensitive outer regions which command maneuvering of the missile to correct the heading error.

In the following two sections, the proposed aerodynamic control, including the missile aerodynamics and the control scheme of operation, and the proposed guidance method will be discussed in detail.

#### Proposed Aerodynamic Control

The proposed method of control is based on a missile configuration having three stable trim points in a usable angle-of-attack range. These trim points are at positive, negative, and zero values of angle of attack. Such a situation is known to exist (ref. 5) and can be determined rationally. In general, such a configuration has low-aspect-ratio wings and tails of equal span with a large ratio of body diameter to span and the wings interdigitated  $45^\circ$ . Such characteristics ease the storage problem of the missile in or on an airplane and, hence, are of great interest. The three stable trim positions result from the changing interference

effects of the wing on the tail as the angle of attack is changed. The extent of the nonlinearity is of course determined by the interference effects at various angles of attack and by the center-of-gravity location.

Figure 1 is a drawing of the missile configuration considered in this paper and is taken from reference 5. For the calculations made herein, the model was scaled to a body diameter of 8 inches and the mass and aerodynamic parameters used are tabulated in table 1. The flight condition is  $M = 1.93$  at an altitude of 30,000 feet; the  $\alpha_{trim}$  values result in  $\pm 10.4g$  maneuvering acceleration. Figure 2 presents the variation of  $C_m$  with  $\alpha$  for this configuration at  $M = 1.62$  and  $M = 1.93$  at a center-of-gravity station of 4.963 inches and was determined from the wind-tunnel results. The pitching-moment coefficient herein is based on the total wing area in one plane and the mean aerodynamic chord of the wing. Because of symmetry only the positive angle-of-attack region is shown. Reference 6 indicates that tests of this scale in the same tunnel compare very favorably with free-flight large-scale results.

The system operates in such a way that the missile is at one of the three trim positions at all times. Thus, this on-off way of using the aerodynamic trim lift leads to the designation, as an aerodynamic-flicker system.

To change from one trim position to another, some form of control torque is required to disturb the missile in the desired direction of  $\alpha$  and  $\delta$  enough to assure reaching the desired trim position; then the control moment is reduced to zero. The simplest form of controlling action would be to use a single control impulse; then, depending on the sense of the input, the control impulse is preset to act in the identical manner for each signal with the direction of control being determined by the sense of the input signal. Another approach that has been tried and will be discussed is a two-pulse control action wherein the control would be first one sign to initiate the angular motion to a new trim position, then the other to stop it, and then zero. Dwell times for a control sequence would be fixed but not necessarily equal. Then, upon receipt of a control signal the same sequence would be used, but the direction of control would be determined by the sense of the input signal.

The possibility of using a servo-actuator programmed in this way upon receipt of an input signal affords a simplification in the operation of the control system and should lead to increased reliability of operation. Although the preceding discussion has indicated the use of an aerodynamic control surface as the means for applying the impulsive torque, in reality, all that is required is some torque applied to the missile which might, of course, come from a metered jet or jet vanes, depending on the application. This step toward simplification of the control system is possible because of the use of a specific aerodynamic characteristic. It is quite probable that other aerodynamic characteristics, not necessarily linear, or preset servo operations can also lead to less complex control equipment.

### Proposed Guidance Method

The proposed method for guidance of the aerodynamic-flicker control missile is basically nonlinear and simple. The target seeker is assumed to have a central dead zone. If the target appears in this dead zone, no command acceleration is required and the missile flies with zero command lift and consequently a straight flight path. If the target appears outside of this dead zone, an acceleration command is given to the missile to reduce the error in heading. As the missile maneuvers and the target again crosses into the dead zone, the command is removed and the missile heads approximately straight toward the target.

This method of guidance is not new inasmuch as similar ideas were applied to glide bombs in World War II. This method is also essentially that used for path corrections in the system of reference 2. The upper part of figure 3 illustrates the two-dimensional graphical relationship between the missile-target line-of-sight angle  $\sigma$ , the missile flight-path angle  $\gamma$ , and the error  $\epsilon = \sigma - \gamma$ . This figure has been drawn with a dead zone of  $10^\circ$  which might, of course, be a design variable in a particular case. Three error situations are shown (figs. 3(a), (b), and (c)), and the command accelerations for each are illustrated in figure 3(d) which is a sketch of command acceleration against error. The basic block diagram of the seeker-missile combination is shown in figure 3(e).

It is also realized that such a guidance method is basically a variation of a pursuit navigation course. As indicated in reference 2 and by the previous discussion, the results of kinematic studies on pursuit navigation, which have shown that infinite rates of turn may be required, have been perhaps overemphasized, and as a result only a meager amount of research has been directed toward such systems. A statement of reference 7 is worthy of reiteration: "However, the extreme practical simplicity of pursuit courses is greatly in their favor, hence one should not decide against their use before considering carefully the extent to which the conclusions of point kinematics can be applied to actual missiles and targets." Indeed, it seems quite probable that variations of the basic pursuit navigation might eliminate those undesirable features predicted by the point kinematic studies of the pure pursuit course.

### RESULTS AND DISCUSSION

The characteristics of the aerodynamic-flicker control system have been primarily studied through the use of analog-computer equipment. Both the REAC and the EASE were used in this study and are essentially identical in operation (see ref. 8). In this work, however, the method of representing the nonlinear variation of  $C_m$  against  $\alpha$  differed in the two setups. In the REAC solutions the nonlinearity was represented on an input-output

table by the actual curve by using wind-tunnel data. In the EASE solutions the nonlinearity was generated by circuits containing diode vacuum tubes. Hence, in this latter situation the nonlinearity is represented by straight lines with relatively sharp breaks. The straight lines used in such a representation were adjusted to have the correct  $\alpha$ -intercepts and the correct slope at the  $\alpha$ -intercepts. This method of representation is shown in figure 2 where the short-dashed line is a work-up of the record of the output of the nonlinear element  $C_m$  for the variation of input  $\alpha$ . The accuracy of the REAC transients was estimated to be approximately 2 percent while those from the EASE were probably about 5 percent.

In the presentation to follow, the free oscillations of the missile are described from transient results and through the use of a phase-plane approach. Finally, the results of two methods of control action in changing trim positions are described, namely the one-pulse method and the two-pulse method.

Assumptions.— In the analysis herein the following assumptions were made regarding the missile and control system operation:

- (1) All the stability derivatives except, of course, the variation of  $C_m$  against  $\alpha$  were considered as constants for the one Mach number used,  $M = 1.93$ .
- (2) Two longitudinal degrees of freedom were assumed, the forward velocity being held constant (ref. 9).
- (3) The control action in all cases was a sharp-edge step.
- (4) Deflection of a trailing-edge flap on the tail surface was considered to apply the control moment to the missile, and the nonlinear stability characteristic was not changed with control deflection (ref. 10).

Data are presented in reference 5 which show that for the present configuration the nonlinear stability characteristic does change with sideslip. This change is not considered herein but does represent a factor that could cause difficulties in a three-dimensional maneuver. The damping in pitch is almost certainly a function of angle of attack, the variation and consequent effect of which is not investigated here. Data in reference 5 also indicate that  $C_{mq} + C_{m\dot{\alpha}}$  probably has a lower value at  $\alpha_{trim}$  values than at  $\alpha = 0$  with a probable increase in this parameter in the region  $0 < \alpha < \alpha_{trim}$ . Such a variation is related, of course, to the interference effects and could be expected; however, the level and shape of the variation with  $\alpha$  is not considered sufficiently well known at the present to warrant inclusion in these results. Possible effects of such variations are discussed in later sections.



## General Description of Free Oscillations

As the missile is required to change from one trim position to another, the oscillatory characteristics of the missile play an important part in the system operation. These characteristics will be discussed in terms of the results of a transient study and the phase-plane approach for the illustration of the system operation.

Transient study.- The dynamical characteristics of the missile are determined herein by two-degree-of-freedom longitudinal response, and the equations for lift and pitching moment are the same as presented in reference 9. From such a consideration the oscillatory characteristics of the aerodynamic-flicker system can be generally described.

Near trim or steady state, the oscillatory features can be determined from a linear analysis, and the period of the oscillations is a function of the slope  $\partial C_m / \partial \alpha$  at trim. The damping at trim remains constant since the lift-curve slope and damping-in-pitch parameters are held constant. Thus, as the steady state is approached a damped oscillation is expected.

In the regions of static instability ( $\partial C_m / \partial \alpha > 0$ ), the dynamic characteristic is an aperiodic divergence. Hence, these regions represent conditions where energy is put into the system, and the system therefore does not tend to stay in these regions.

In general, therefore, the transient characteristics can be summarized as follows:

- (1) Some form of control disturbs the missile from a steady state.
- (2) At the time that control actuation is brought back to zero, certain conditions of  $\alpha$  and  $\dot{\theta}$  exist.
- (3) Depending on these values, which might be considered as initial conditions for the free oscillations, one of the three trim or steady-state positions is finally reached.

As an illustration of this last statement, consider a control deflection applied to the missile for a very short time where initially  $\alpha = \dot{\theta} = 0$ . Upon release of the control, the angle of attack and pitching velocity are still small and the linear range about  $\alpha = 0$  is not traversed, and the motion therefore returns to the central trim position,  $\alpha = \dot{\theta} = 0$ . If the control is held longer, the central region is traversed and possibly the unstable region is entered in such a way that when the control is brought to zero, the transient proceeds to the positive  $\alpha_{\text{trim}}$  position. If the control is held still longer before release and the outer stable region is well entered, it is possible that the motion will not remain about the  $\alpha_{\text{trim}}$  position but will enter the unstable region and finally stabilize at the central trim position.



There are three stable regions of angle of attack separated by two unstable regions. It is impossible for this system to develop a constant hunting oscillation (see ref. 3, p. 227); the free oscillations always damp to a steady-state position. Reference 9 discusses a similar situation where the pitching-moment curve results in a system having two stable regions separated by an unstable one, and remarks similar to reference 3 apply.

To illustrate the effect of control-surface-actuation times on the transient responses of the aerodynamic-flicker system, figure 4 is presented in which the initial conditions at  $\alpha_{trim}$  are considered. The six transients in this figure illustrate how the various trim positions can be successively approached. These are actual records taken from the EASE analysis and depend on the assumption that the nonlinear stability characteristic is unaffected by control deflection.

Phase-plane approach.- The phase-plane method used extensively in nonlinear mechanics (refs. 3 and 4) affords further insight into the details of the transient characteristics. Reference 9 also discusses in some detail the methods used to determine the phase-plane characteristics derived herein. Briefly, the phase plane is immediately applicable to those dynamical situations, linear or nonlinear, where two variables will completely define the system operation. This is the case for the two-degree-of-freedom longitudinal response of a missile since angle of attack  $\alpha$  and pitching velocity  $\dot{\theta}$  completely characterize the motion. Thus a plot of  $\dot{\theta}$  against  $\alpha$  could represent the successive states that the missile might take and is therefore a logical phase plane for this study. The trajectories in the phase plane represent the successive states that the motion takes following a given set of initial conditions although the time at which the states exist is not usually known.

For the analysis herein, the nonlinear curve of  $C_m$  against  $\alpha$  was approximated by straight lines with the correct intercepts and correct slopes at the intercepts. Actually this made the system representation linear in the several regions, or "piecewise linear." Families of isoclines, where each isocline is the locus of points where the trajectories have a given slope (ref. 3, pp. 10 and 248), were drawn for the several regions of angle of attack. From an inspection of the isoclines in the regions of instability, it was possible to denote a point where the motion would go back to the central trim position but where a very small variation to one side of this point would result in motion to an outer trim position. From this point a boundary could be determined between those regions where the motion would go to one trim position or another. Figure 5 is a phase-plane diagram for the missile considered herein for the conditions previously noted, and the boundary curves are shown. It is remembered that at the two outer trim positions the missile is pulling lift and hence has a steady-state value of pitching velocity  $\dot{\theta}$ .

The cross-sectional and dotted regions of figure 5 represent those regions wherein an initial condition would result in a free oscillation and finally in the steady-state positions shown. The clear regions between the other shaded regions applies to those conditions which result in a steady-state position at the origin. The arrows indicate how the various regions are traversed in approaching the steady state. Since the motion near steady state is oscillatory, the trajectories themselves encircle the trim value as time increases. A trajectory is shown (dashed curve) in figure 5, where the initial conditions were  $\alpha = 8.0^\circ$ ,  $\dot{\theta} = -0.65$  radian per second, and the resultant motion is oscillatory about  $\alpha = \dot{\theta} = 0$ . A trajectory is also shown with the initial conditions of point a.

The solid curve starting at the  $\alpha_{\text{trim}}$  point is the trajectory for the case when  $\delta = 5^\circ$ . If the control is held deflected to points b, d, or f, the motion goes to steady state at  $\alpha = \dot{\theta} = 0$ . Similarly, if the control is held to points a or e, the motion goes to  $+\alpha_{\text{trim}}$ ; and if held to point c, the motion goes to  $-\alpha_{\text{trim}}$ . If the control were held even longer, a point would be reached to the left of the figure where the motion would go to the  $-\alpha_{\text{trim}}$  position.

The points a to f are the actual conditions at the time the control is returned to zero from the cases shown in figure 4. Thus, the transients of figure 4 directly correlate with the information determined from the phase plane of figure 5.

More accurate results may be obtained from a slightly more laborious phase-plane analysis. This would require an analytic expression for the curve of  $C_m$  against  $\alpha$  instead of a representation by straight lines. This is the approach generally taken in nonlinear mechanics (refs. 4, 5, and 9) for which the methods are most useful. Following this initial analytical step, the graphical analysis could still be used.

#### One-Pulse Method of Control Operation

It has been indicated in a previous section and in figure 4 that a single control pulse, or impulse, is sufficient to change the missile from one trim position to another. The characteristics of this method of control are now discussed.

First, it is realized that there is a minimum control moment required to change from one trim position to another. If the control moment is less than this value, the stability of the configuration about the trim value from which the motion was started will be sufficient to balance out the control moment. Thus, no matter how long this control moment is applied, neither of the other two trim positions could be attained.

For control moments above this minimum, there are definite control-actuation times, or dwell times, that result in changes from one trim position to another; or, in other words, certain moment impulses result in changes in trim position. For example, in changing from  $\alpha = 0$  to positive  $\alpha_{\text{trim}}$ , minimum actuation time  $\Delta t$  just allows the missile to go to  $\alpha_{\text{trim}}$ . For  $\Delta t$  less than this minimum, the motion oscillates back to  $\alpha = 0$ . A maximum value of  $\Delta t$  also results in the motion remaining finally at  $\alpha_{\text{trim}}$ . If this maximum  $\Delta t$  is exceeded, the motion again goes back to  $\alpha = 0$ . Thus, a range of time between these minimum and maximum values at  $\Delta t$  results in the desired change from  $\alpha = 0$  to  $\alpha_{\text{trim}}$ . Similarly, if initially the missile is at  $\alpha_{\text{trim}}$ , a range of time between minimum and maximum values of  $\Delta t$  results in a change from  $\alpha_{\text{trim}}$  to  $\alpha = 0$ .

Figure 6 is a summary of the actuation times  $\Delta t$ , both maximum and minimum, as a function of the control moment coefficient  $C_m(\delta)$ . The actual variation of  $C_m$  against  $\alpha$  was used to obtain these results from the REAC. The region between the upper dashed line and the lower solid line is values of  $\Delta t$  that are satisfactory for operation from either  $\alpha = \delta = 0$  to  $\alpha_{\text{trim}}$  or from  $\alpha_{\text{trim}}$ ,  $\delta_{\text{trim}}$  to  $\alpha = 0$ . Therefore, this region satisfies those conditions which should lead to a simplification of the servomotor, because, for a given control moment, the servo would have the same actuation time for either sense of operation. For the trailing-edge flap considered, the deflection in degrees is also indicated by the lower abscissa scale. This region presents a very close, though probably not impossible, tolerance on the servo operation (e.g., at  $\delta = 6^\circ$ , the value of  $\Delta t$  must be between 0.175 second and 0.211 second). Since this tolerance is so very close, the possible variation of the damping-in-pitch parameters with angle of attack would probably be important for the one-pulse method of control.

For such a one-pulse method to be useful, it would have to apply for some range of Mach number and altitude. Inasmuch as this  $\Delta t$  region is very small, additional requirements might make such an approach impossible. For the cases studied, however, it is possible to find values which are satisfactory for both  $M = 1.93$  and  $M = 1.62$ .

Changes in trim from  $+\alpha_{\text{trim}}$  to  $-\alpha_{\text{trim}}$  have not been tabulated because the system operation would never require such a change in the guidance scheme of control as previously discussed.

It is realized that the control motion in actuality would not be as abrupt a step as those of the analog method but would probably be more trapezoidal in shape. However, if the trapezoidal pulse had the same area as the rectangular pulse, the results would probably be essentially identical.

## Two-Pulse Method of Control Operation

In addition to the single-pulse method for changing from one trim position to another, it is possible to use a two-pulse method of control actuation and to improve considerably the response time of the missile. Instead of moving the control from zero to a deflection and later back to zero, the two-pulse method requires a control motion from zero to a deflection, then to an equal but opposite deflection, and later back to zero. The two-pulse method uses the second pulse to retard the motion and can result in no overshoot, or smaller overshoot, of the desired trim position.

In figure 7(a) the deadbeat response in changing from  $\alpha = 0$  to  $\alpha_{trim}$  is shown with the control action which produced this change and represents the minimum response time (0.5 second) possible with the two-pulse method where  $\delta = \pm 5^\circ$ . It is noted in figure 7(b) that the same control actuation produces a somewhat oscillatory change when used from  $\alpha_{trim}$  to  $\alpha = 0$ . (Compare the time-scale change of figs. 7 and 8 with that of fig. 4.) In figure 7(c) the deadbeat response from  $\alpha_{trim}$  to  $\alpha = 0$  is shown with the corresponding control actuations. Again, this is the minimum response time (0.47 second) possible with the two-pulse method when  $\delta = \pm 5^\circ$ . Figure 7(d), however, shows that this control sequence is not satisfactory when the initial condition is  $\alpha = 0$  because  $\alpha_{trim}$  is not reached.

Figure 8 shows a compromise sequence which gives about the same response times for both initial conditions for a total control-actuation time of  $1/2$  second. Additional EASE cases such as these indicate a much wider tolerance in actuation times allowable to assure satisfactory operation from  $\alpha = 0$  to  $\alpha_{trim}$  and from  $\alpha_{trim}$  to  $\alpha = 0$ . The more critical dwell time is that of the first pulse because it must be long enough to assure the motion from  $\alpha = 0$  to  $\alpha_{trim}$ ; the duration of the second pulse is much less critical. For example, in 35 additional cases which quickly surveyed the dwell times required for the two-pulse method to operate satisfactorily when  $\delta = \pm 5^\circ$ , a value of  $\Delta t$  of 0.30 second was about the minimum time required for the first pulse to go from  $\alpha = 0$  to  $\alpha_{trim}$ . The range  $0.30 \text{ second} < \Delta t < 0.40 \text{ second}$  proved very satisfactory for the first pulse in both directions of operation (i.e., from  $\alpha = 0$  to  $\alpha_{trim}$  and from  $\alpha_{trim}$  to  $\alpha = 0$ ). The range of value of  $\Delta t$  for the second pulse for either direction of operation was from  $0.08 \text{ second} < \Delta t < 0.28 \text{ second}$ . It is believed that these are not the complete ranges for either direction of change since the absolute limits of operation for the second pulse at a given dwell time for the first pulse was not established. It is emphasized that in this section, as in the previous one, reference to  $\alpha_{trim}$  necessarily means that  $\theta = \theta_{trim}$ .

The two-pulse method therefore has the distinct advantages of having greater tolerances on the required dwell times and shorter response times with less oscillation. The greater tolerance would tend to make the effects of damping-in-pitch variations with angle of attack, Mach number, and altitude changes less critical for the two-pulse method of control. This method would require a three-position fixed-sequence servo for control operation.

### Simplified Trajectory Result

In order to illustrate the operation of a missile in the guidance scheme previously discussed, a simplified trajectory analysis was made. The missile-to-target velocity ratio was assumed to be 2 with a missile speed of 1,800 feet per second, and both velocities were considered constant. The initial missile-target range was 25,000 feet in a beam-attack situation, and the target was assumed to be nonmaneuvering. The seeker dead zone was  $\pm 5^\circ$ , the command accelerations were  $\pm 10g$ , and the target seeker was always aligned along the velocity vector. The missile characteristics were simplified in that the missile acceleration was considered to occur instantaneously,  $1/2$  second after the acceleration command was given. Obviously this only takes care of the approximate response time of the missile and does not consider the oscillatory characteristics of the missile attitude and the way in which the seeker is mounted. This simplified trajectory led to a miss distance of 12 feet. While, admittedly, this represents an oversimplification of the actual situation, such a scheme of guidance affords much promise toward simplicity and reliability and is worthy of further considerations.

### CONCLUDING REMARKS

The use of a missile configuration having three stable trim positions at positive, negative, and zero angles of attack has been discussed as a possible means of simplifying the required control system. Such a configuration has been studied, and the required servo operation for changing the missile from one trim position to another is presented for one illustrative flight condition from analog computer results. Two control methods have been illustrated; namely, the one-pulse method, where a constant actuation time is used when a plus or minus control moment is applied to change trim positions, and the two-pulse method, where a control sequence of first one deflection, then an equal but opposite deflection, and finally zero deflection are employed. The two-pulse method was shown to have greater tolerances in the actuation times required of the servomotor with resultant motions having shorter response times with less oscillations, thus leading to a system capable of handling a wider variety of flight conditions. This operation of the missile at either of the three trim

positions at all times has led to the designation as an aerodynamic-flicker system. The use of preset control actuations which operate identically in sequence for all commands affords the simplification of the internal control equipment. The use of such a system in a guidance situation has been discussed, and indications are that simplified guidance equipment can also be employed.

Langley Aeronautical Laboratory,  
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TABLE I

MISSILE CHARACTERISTICS AND STABILITY DERIVATIVES AT  $M = 1.93$ 

|                                                                                                     |        |
|-----------------------------------------------------------------------------------------------------|--------|
| Mass, slugs . . . . .                                                                               | 5.05   |
| Wing area, $S$ , sq ft . . . . .                                                                    | 1.55   |
| Mean aerodynamic chord, $\bar{c}$ , ft . . . . .                                                    | 1.24   |
| Moment of inertia, slug-ft <sup>2</sup> . . . . .                                                   | 31.3   |
| $C_{L_\alpha}$ , $\partial C_L / \partial \alpha$ , per radian . . . . .                            | 2.95   |
| $C_{m_q}$ , $\partial C_m / \partial \frac{\dot{\theta} c}{2V}$ , per radian . . . . .              | -26.10 |
| $C_{m_{\dot{\alpha}}}$ , $\partial C_m / \partial \frac{\dot{\alpha} c}{2V}$ , per radian . . . . . | -1.95  |
| $C_{m_\delta}$ , $\partial C_m / \partial \delta$ , per radian . . . . .                            | -.61   |
| At 30,000-ft altitude:                                                                              |        |
| Velocity, $V$ , ft/sec . . . . .                                                                    | 1,925  |
| Dynamic pressure, $q$ , lb/sq ft . . . . .                                                          | 1,650  |

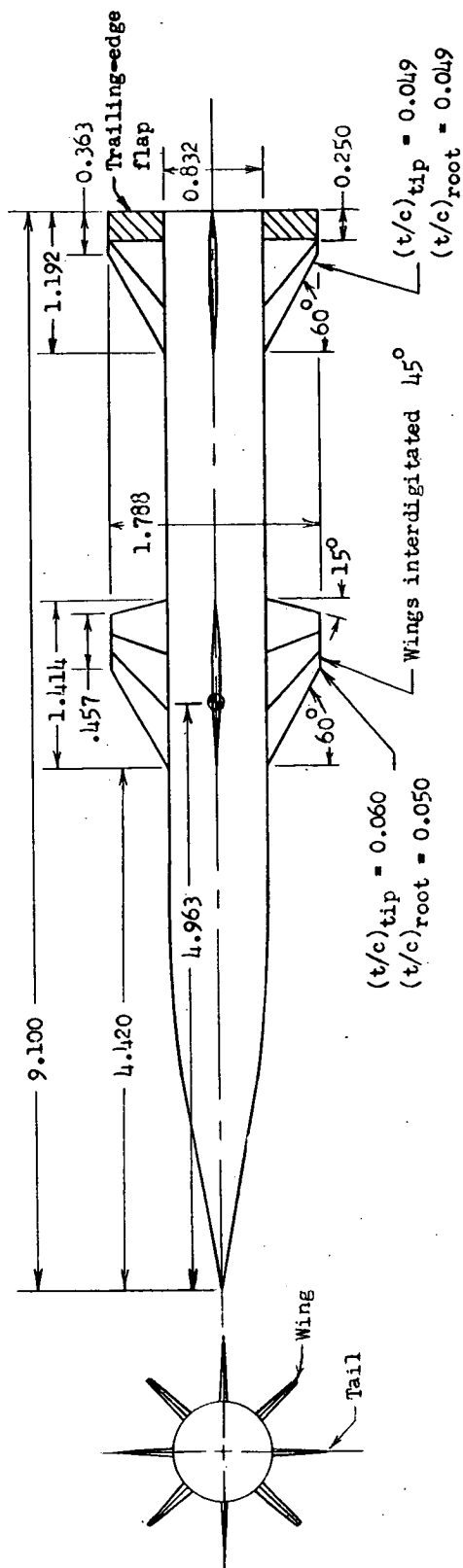


Figure 1.- Sketch of configuration having three stable trim positions.  
 (Dimensions are in inches.)

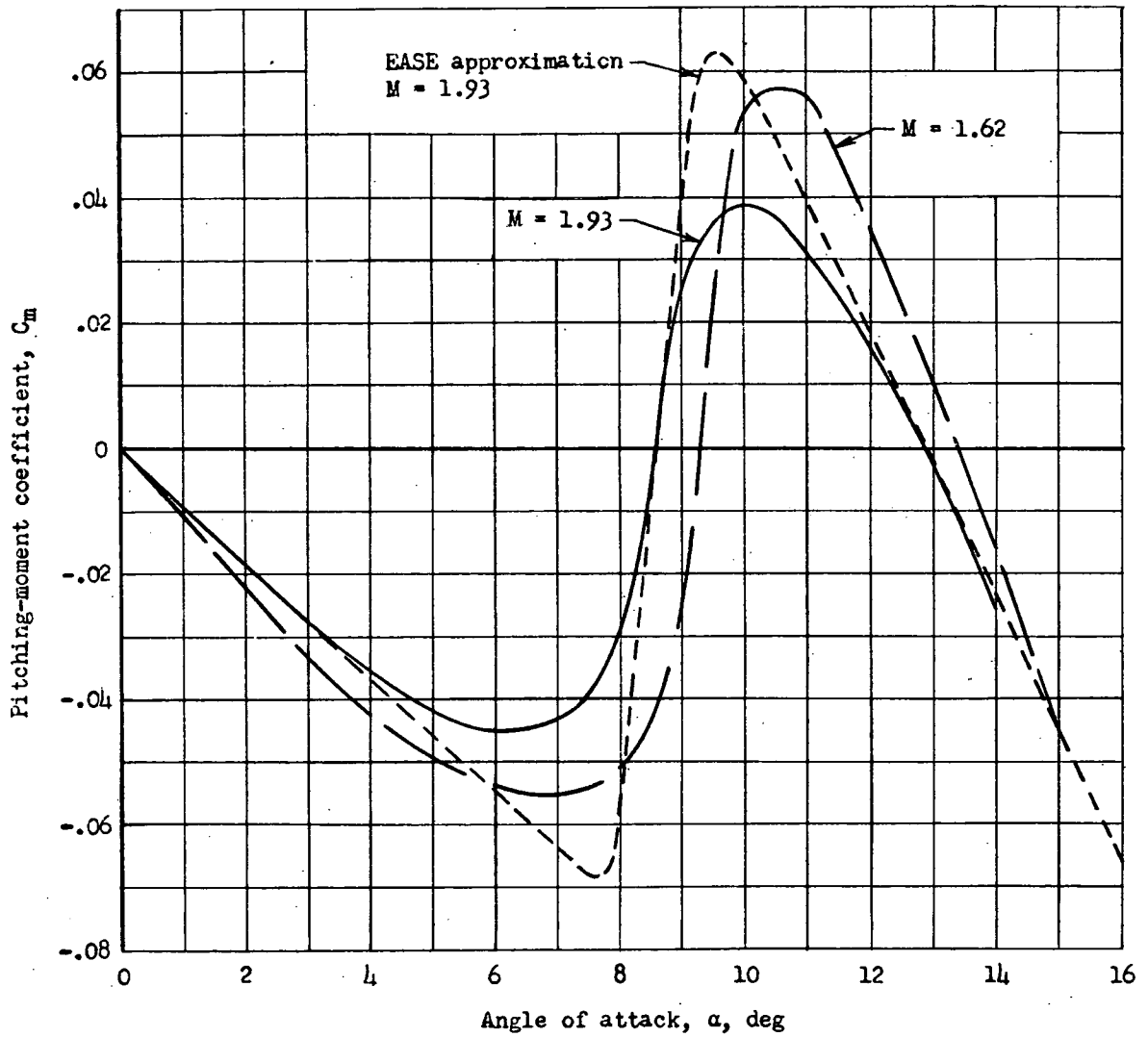


Figure 2.- Pitching-moment curves obtained from wind-tunnel data (ref. 5).

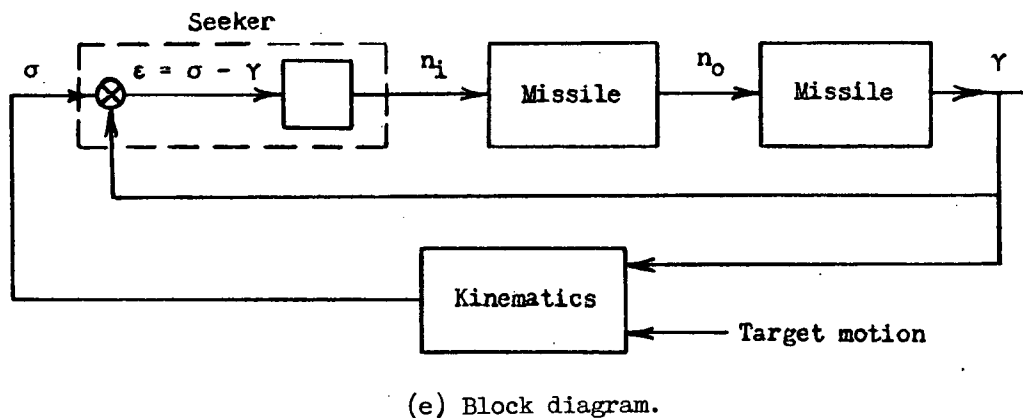
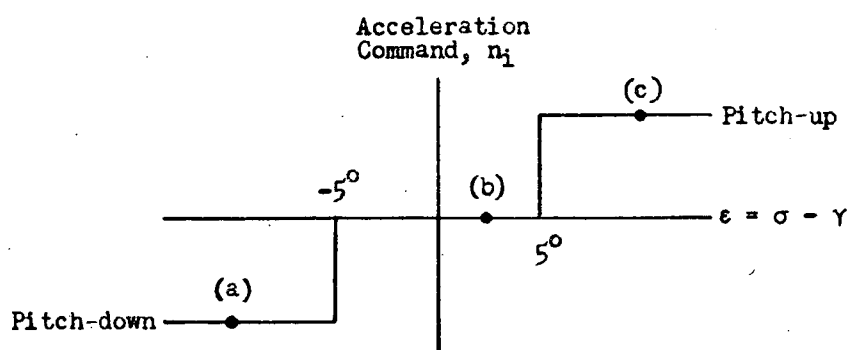
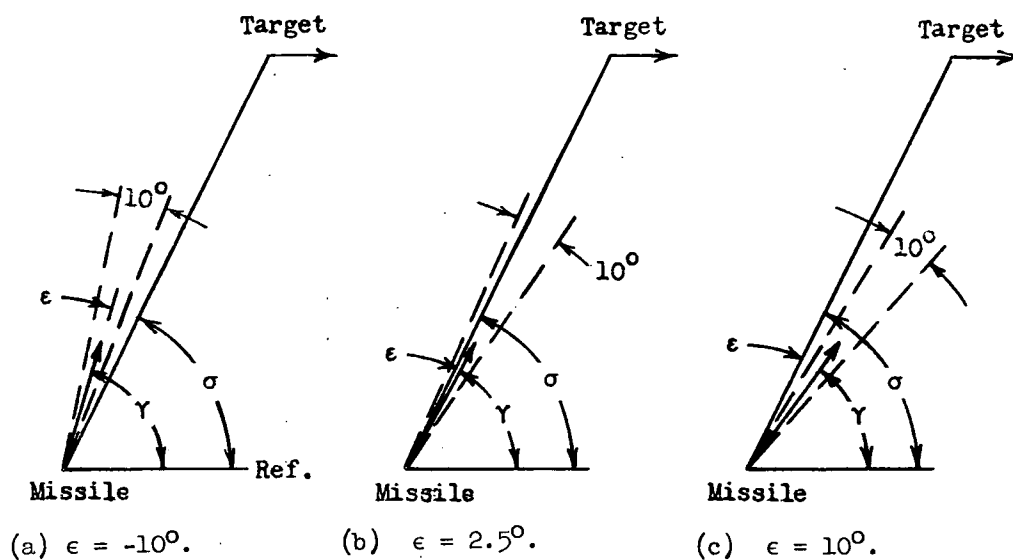


Figure 3.- Description of seeker operation in guidance problem.

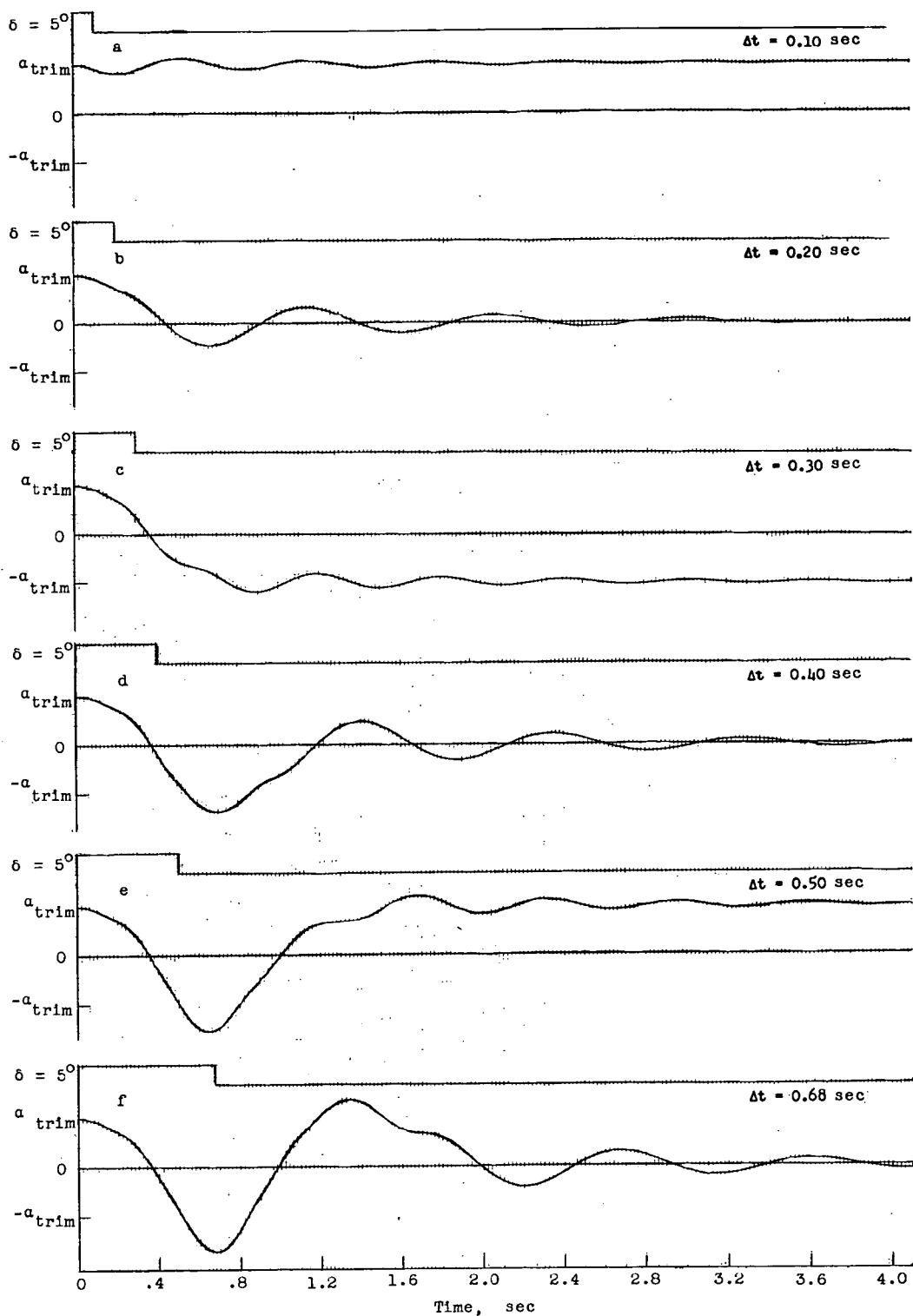


Figure 4.- EASE records showing variation of transient response of missile with changes in control-surface-actuation time.

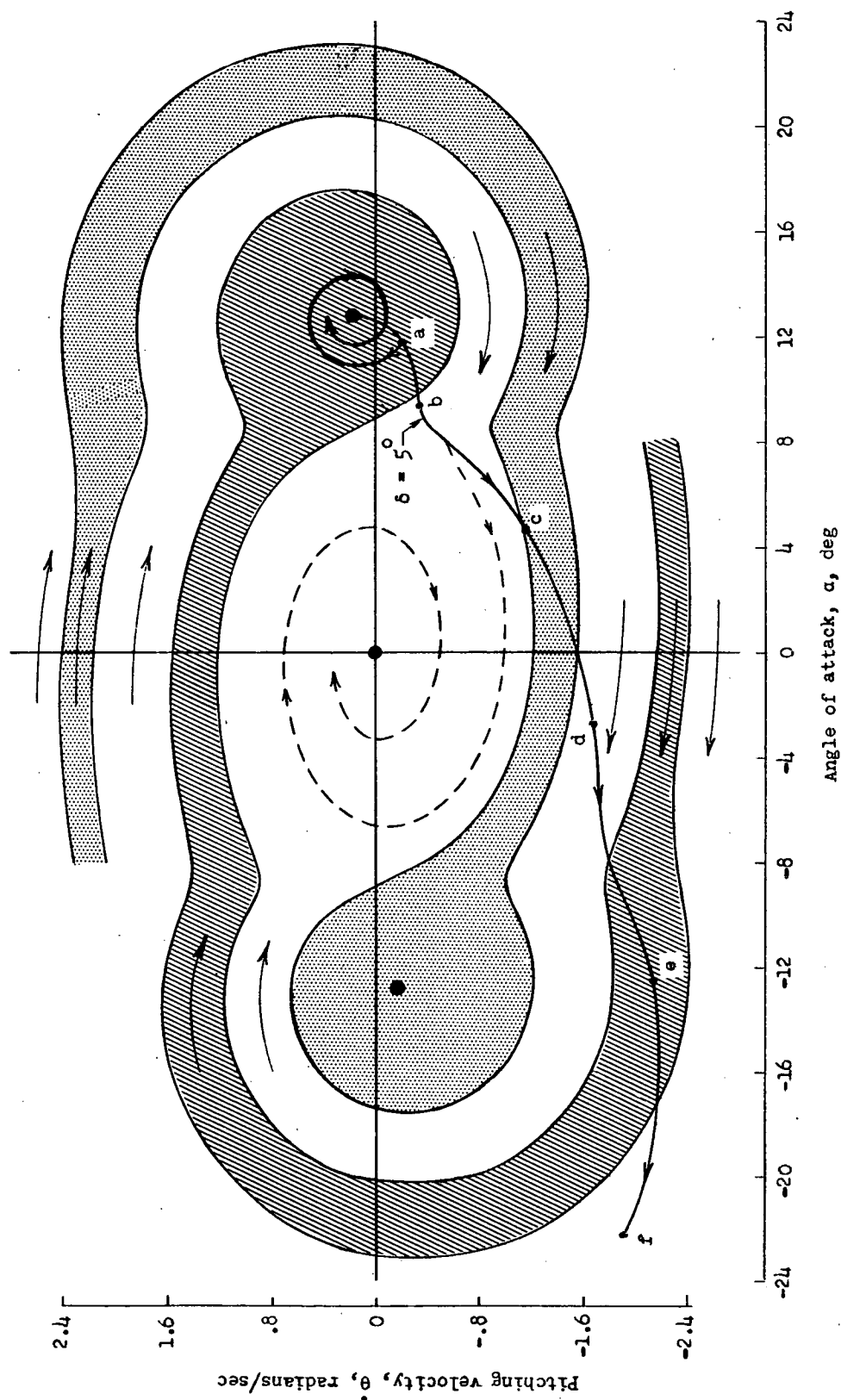


Figure 5.-- Phase-plane diagram illustrating operation of three trim positions of missile based upon straight-line approximations of pitching-moment curve. (Motions for initial conditions in the various regions result in the steady state for that region.)

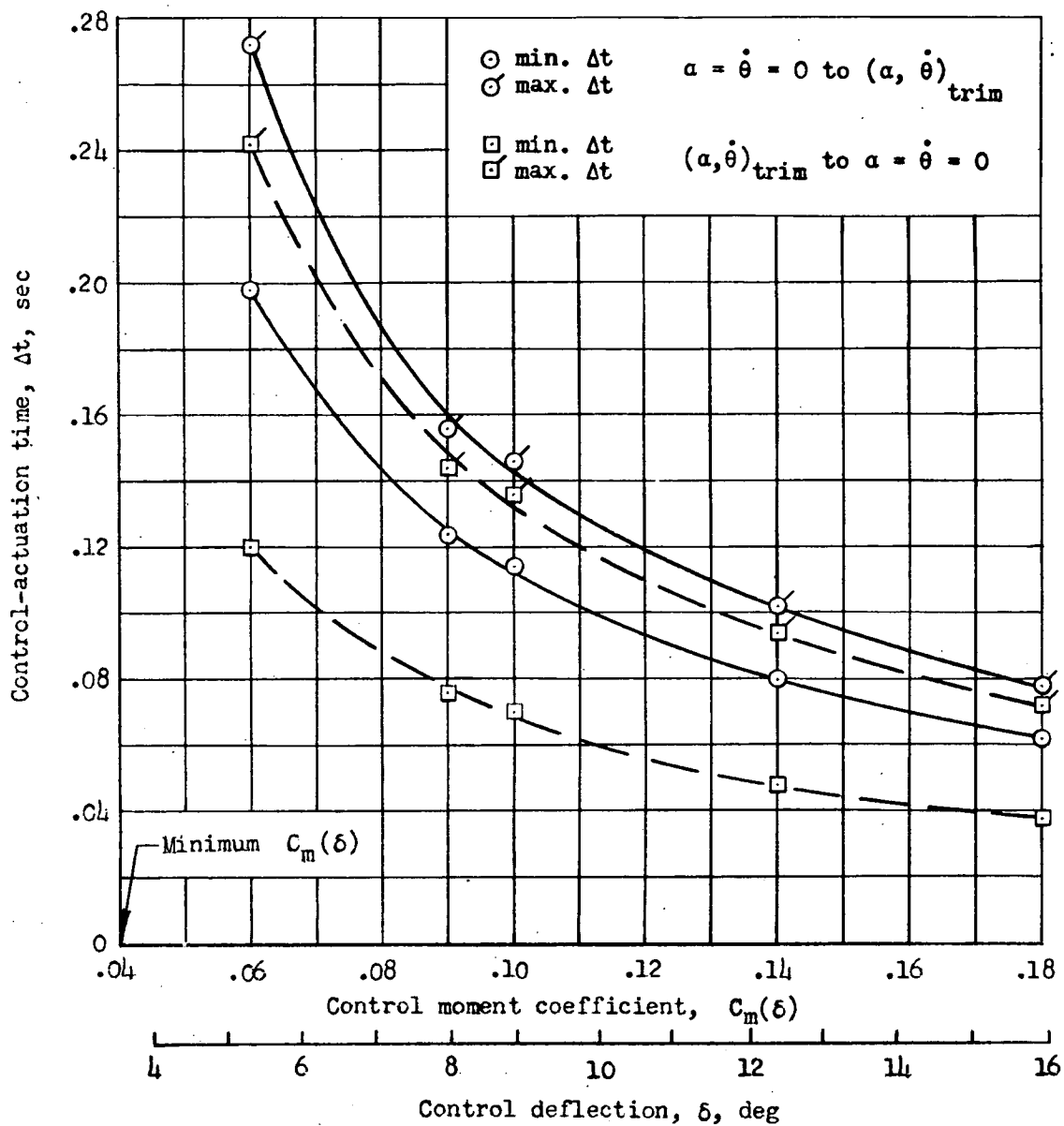


Figure 6.- Actuation times for one-pulse control method as determined from REAC data.

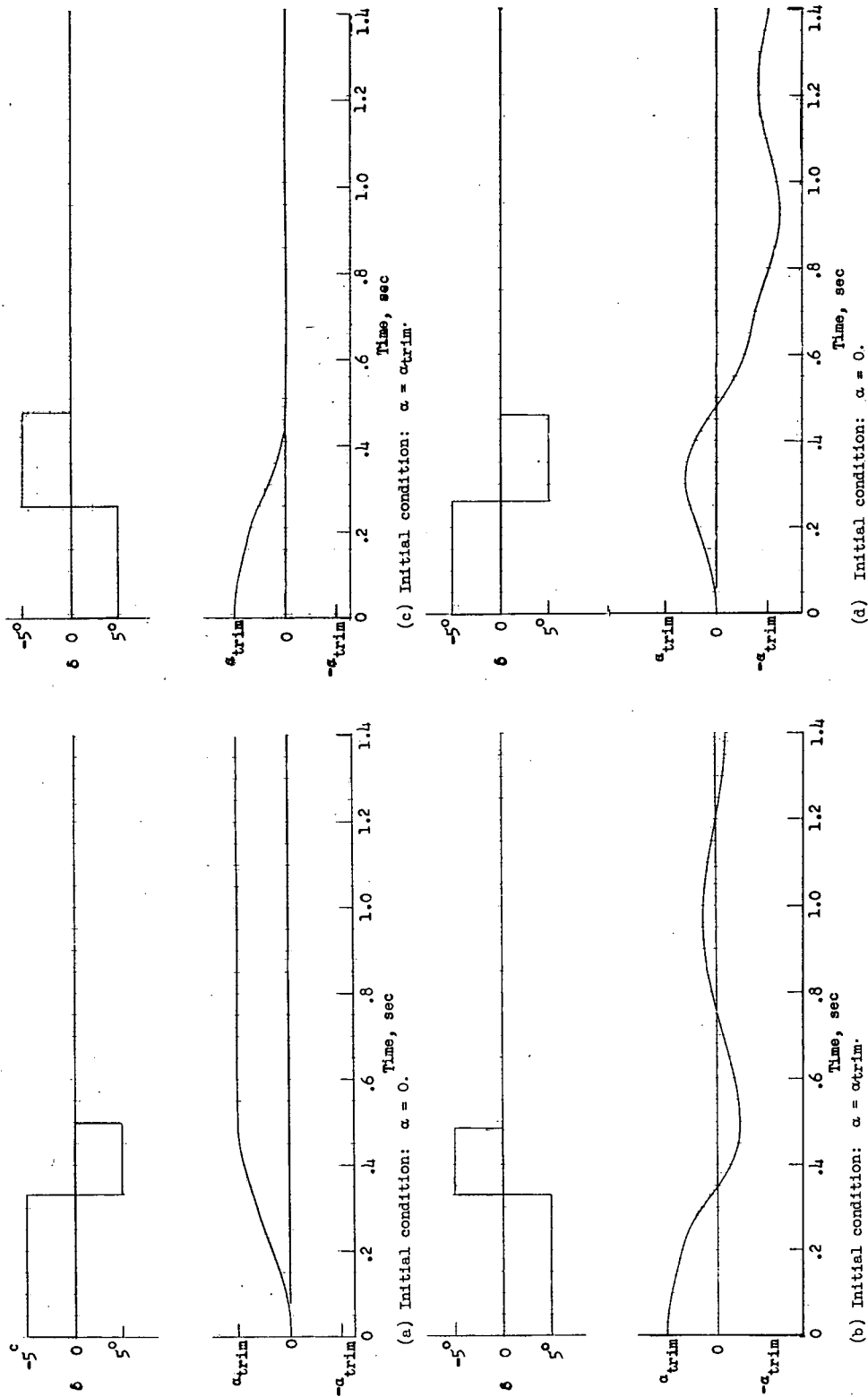


Figure 7.- EASE records of transient responses to two-pulse control method.  
 $\alpha = 0$  and  $\alpha_{trim}$ ;  $\theta = 0$  and  $\theta_{trim}$ .



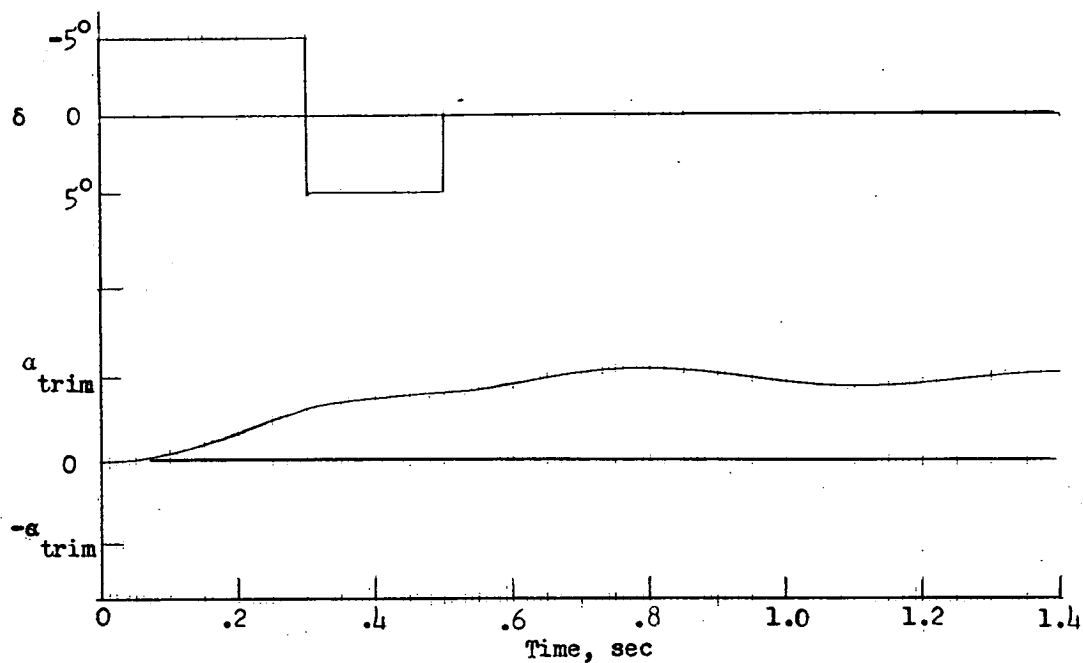
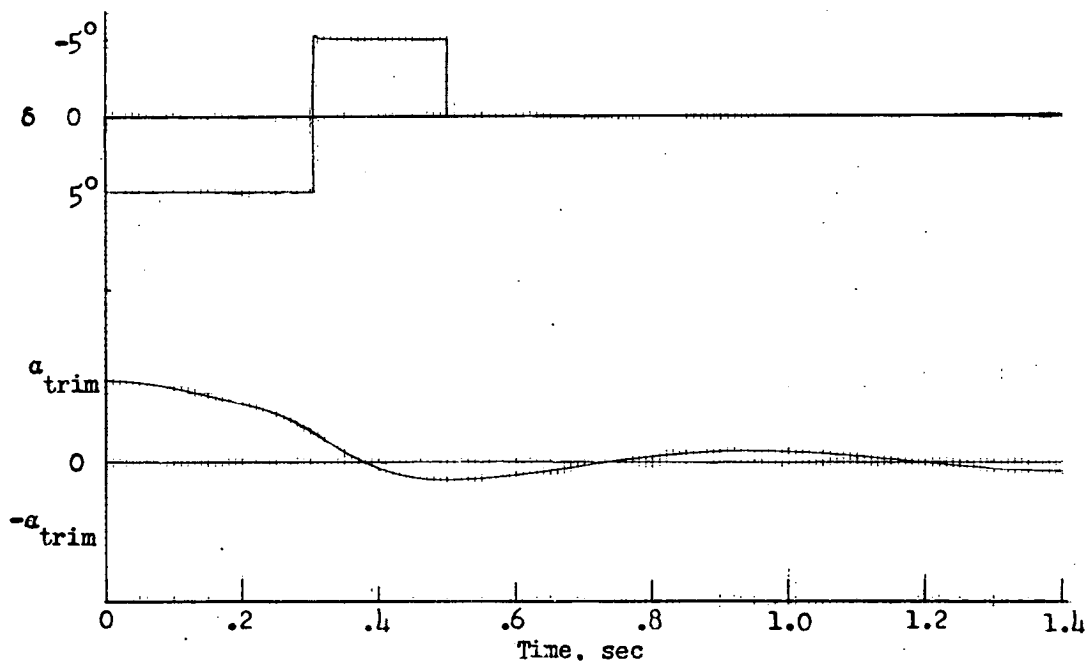
(a) Initial condition:  $\alpha = 0$ .(b) Initial condition:  $\alpha = \alpha_{trim}$ .

Figure 8.- EASE records of transient responses to two-pulse control method.  
Total control-actuation time is  $1/2$  second.